Ch.4. Transmission Line Parameters

4.1. Transmission Line Design Considerations:

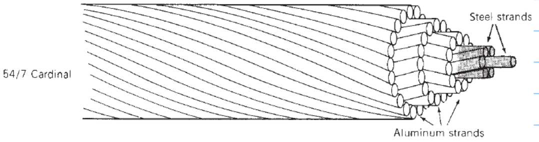
* Transmission lines consist of:

① Conductors:

Types:

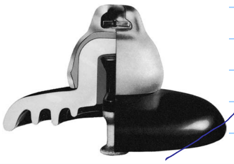
Aluminum conductor, steel re-inforced	(ACSR)
All - Aluminume conductor	(AAC)
All - Aluminume - alloy conductor	(AAAC)
Aluminum conductor, Alloy-reinforced	(ACAR)
Aluminum-clad steel conductor	(Alumowed)

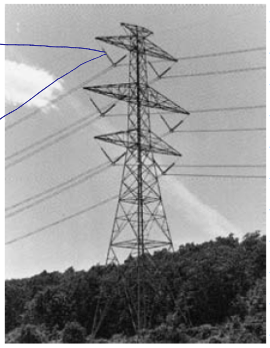
Bundle: more than one conductor per phase to control corona and reduce the electric field strength.



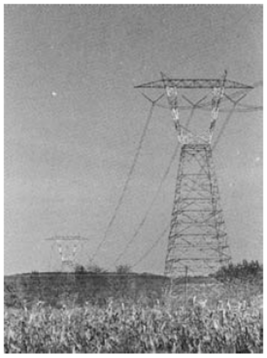
② Insulators:

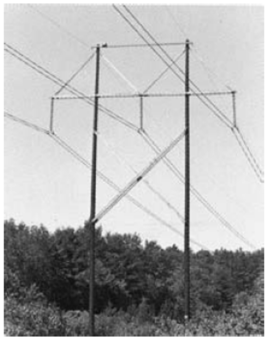
Nominal Voltage	Suspension Insulator String		Shield Wires		
(kV)	Number of Strings per Phase	Number of Standard Insulator Discs per Suspension String	Type	Number	Diameter (cm)
69	1	4 to 6	Steel	0, 1 or 2	
138	1	8 to 11	Steel	0, 1 or 2	
230	1	12 to 21	Steel or ACSR	1 or 2	1.1 to 1.5
345	1	18 to 21	Alumoweld	2	0.87 to 1.5
345	1 and 2	18 to 21	Alumoweld	2	0.87 to 1.5
500	2 and 4	24 to 27	Alumoweld	2	0.98 to 1.5
500	2 and 4	24 to 27	Alumoweld	2	0.98 to 1.5
765	2 and 4	30 to 35	Alumoweld	2	0.98





③ Support Structure: & ④ Shield Wires:





Thereafter, transmission-line design is based on optimization of electrical, mechanical, environmental, and economic factors.

4.2 Resistance:

$$R_{dc,T} = \frac{\rho_T l}{A} \quad \Omega$$

where ρ_T = conductor resistivity at temperature T

l = conductor length

A = conductor cross-sectional area

$D \text{ in} \xrightarrow{\times 1000} d \text{ mil}$, $1 \text{ cmil} = \frac{\pi}{4} \text{ sq mil}$

$$A = \left(\frac{\pi}{4} D^2 \text{ in.}^2\right)\left(1000 \frac{\text{mil}}{\text{in.}}\right)^2 = \frac{\pi}{4}(1000 \text{ D})^2 = \frac{\pi}{4} d^2 \quad \text{sq mil}$$

or

$$A = \left(\frac{\pi}{4} d^2 \text{ sq mil}\right)\left(\frac{1 \text{ cmil}}{\pi/4 \text{ sq mil}}\right) = d^2 \quad \text{cmil}$$

Conductor resistance depends on the following factors:

Read Ex 4.1

1. Spiraling → Stranded ⇒ + 1% ~ 2% longer
2. Temperature → $\rho_{T2} = \rho_{T1}\left(\frac{T_2 + T}{T_1 + T}\right)$
3. Frequency ("skin effect") → $R_{ac} = \frac{P_{loss}}{|I|^2}$ Ω
4. Current magnitude—magnetic conductors

4.3 Conductance:

- Caused by insulator leakage current and corrona.
- Very small ⇒ neglegable.

4.4. Inductance: Solid Cylindrical Conductor:

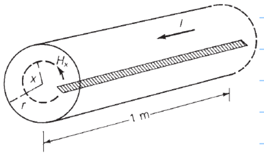
The inductance of a magnetic circuit that has a constant permeability μ can be obtained by determining the following:

1. Magnetic field intensity H, from Ampere's law
2. Magnetic flux density B ($B = \mu H$)
3. Flux linkages λ
4. Inductance from flux linkages per ampere ($L = \lambda/I$)

① Internal inductance: $x < r$

FIGURE 4.6

Internal magnetic field of a solid cylindrical conductor



$$\oint H_{\text{tan}}\, dl = I_{\text{enclosed}}$$

$$H_x(2\pi x) = I_x \quad \text{for } x < r \longrightarrow H_x = \frac{I_x}{2\pi x} \quad \text{A/m}$$

$$I_x = \left(\frac{x}{r}\right)^2 I \quad \text{for } x < r \longrightarrow H_x = \frac{xI}{2\pi r^2} \quad \text{A/m}$$

$$B_x = \mu_0 H_x = \frac{\mu_0 x I}{2\pi r^2} \quad \text{Wb/m}^2$$

$$d\Phi = B_x\, dx \quad \text{Wb/m} \;, \; d\lambda = \left(\frac{x}{r}\right)^2 d\Phi = \frac{\mu_0 I}{2\pi r^4} x^3\, dx \quad \text{Wb-t/m}$$

$$\lambda_{\text{int}} = \int_0^r d\lambda = \frac{\mu_0 I}{2\pi r^4}\int_0^r x^3\, dx = \frac{\mu_0 I}{8\pi} = \frac{1}{2}\times 10^{-7} I \quad \text{Wb-t/m} \longrightarrow \text{L}_{\text{int}} = \frac{\lambda_{\text{int}}}{I} = \frac{\mu_0}{8\pi} = \frac{1}{2}\times 10^{-7} \quad \text{H/m}$$

② External inductance: $x > r$

$$H_x(2\pi x) = I \longrightarrow H_x = \frac{I}{2\pi x} \quad \text{A/m} \qquad x > r$$

Outside the conductor, $\mu = \mu_0$ and

$$B_x = \mu_0 H_x = (4\pi \times 10^{-7})\frac{I}{2\pi x} = 2 \times 10^{-7}\frac{I}{x} \quad \text{Wb/m}^2$$

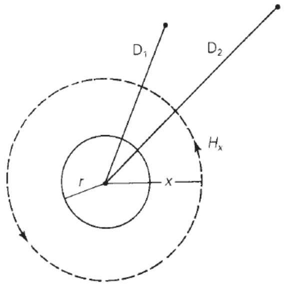
$$d\Phi = B_x\, dx = 2 \times 10^{-7}\frac{I}{x}\, dx \quad \text{Wb/m}$$

Since the entire current I is linked by the flux outside the conductor,

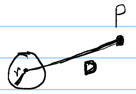
$$d\lambda = d\Phi = 2 \times 10^{-7}\frac{I}{x}\, dx \quad \text{Wb-t/m}$$

$$\lambda_{12} = \int_{\text{D}_1}^{\text{D}_2} d\lambda = 2 \times 10^{-7} I \int_{\text{D}_1}^{\text{D}_2} \frac{dx}{x}$$

$$= 2 \times 10^{-7} I \ln\left(\frac{\text{D}_2}{\text{D}_1}\right) \quad \text{Wb-t/m}$$



$$L_{12} = \frac{\lambda_{12}}{I} = 2 \times 10^{-7} \ln\left(\frac{D_2}{D_1}\right) \quad \text{H/m}$$



If $D_1 = r$, $D_2 = D$: λ_P = internal flux + external flux.

$$\lambda_P = \frac{1}{2} \times 10^{-7} I + 2 \times 10^{-7} I \ln \frac{D}{r}$$

$\left(\frac{1}{2} = 2 \ln e^{\frac{1}{4}}\right)$ identity leads to:

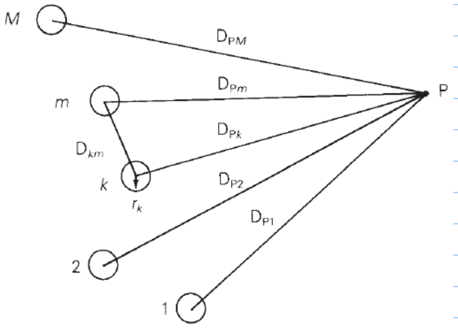
$$\lambda_P = 2 \times 10^{-7} I \left(\ln e^{1/4} + \ln \frac{D}{r} \right)$$

$$= 2 \times 10^{-7} I \ln \frac{D}{e^{-1/4} r}$$

$$= 2 \times 10^{-7} I \ln \frac{D}{r'} \quad \text{Wb-t/m}$$, $r' = e^{-\frac{1}{4}} r = 0.7788\, r$

$$L_P = \frac{\lambda_P}{I} = 2 \times 10^{-7} \ln\left(\frac{D}{r'}\right) \quad \text{H/m}$$

③ Array of Conductors:



$$\lambda_{kPk} = 2 \times 10^{-7} I_k \ln \frac{D_{Pk}}{r'_k}$$

$$\lambda_{kPm} = 2 \times 10^{-7} I_m \ln \frac{D_{Pm}}{D_{km}}$$

$$\lambda_{kP} = \lambda_{kP1} + \lambda_{kP2} + \cdots + \lambda_{kPM} = 2 \times 10^{-7} \sum_{m=1}^{M} I_m \ln \frac{D_{Pm}}{D_{km}}$$

$$\lambda_{kP} = 2 \times 10^{-7} \sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} + 2 \times 10^{-7} \sum_{m=1}^{M} I_m \ln D_{Pm}$$

$$\lambda_{kP} = 2 \times 10^{-7} \left[\sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} + \sum_{m=1}^{M-1} I_m \ln D_{Pm} + I_M \ln D_{PM} \right]$$

$$I_1 + I_2 + \cdots + I_M = \sum_{m=1}^{M} I_m = 0 \longrightarrow I_M = -(I_1 + I_2 + \cdots + I_{M-1}) = -\sum_{m=1}^{M-1} I_m$$

$$\lambda_{kP} = 2 \times 10^{-7}\left[\sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} + \sum_{m=1}^{M-1} I_m \ln D_{Pm} - \sum_{m=1}^{M-1} I_m \ln D_{PM}\right]$$

$$= 2 \times 10^{-7}\left[\sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} + \sum_{m=1}^{M-1} I_m \ln \frac{D_{Pm}}{D_{PM}}\right]$$

Now, let λ_k equal the total flux linking conductor k out to infinity. That is, $\lambda_k = \lim_{P\to\infty} \lambda_{kP}$. As $P \to \infty$, all the distances D_{Pm} become equal, the ratios D_{Pm}/D_{PM} become unity, and $\ln(D_{Pm}/D_{PM}) \to 0$. Therefore, the second summation in (4.4.29) becomes zero as $P \to \infty$, and

$$\lambda_k = 2 \times 10^{-7} \sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} \quad \text{Wb-t/m}$$

4.5 Inductance: 1ϕ two-wire line and 3ϕ three-wire line with equal phase spacing:

① 1ϕ two-wire line:

$$\lambda_x = 2 \times 10^{-7}\left(I_x \ln \frac{1}{D_{xx}} + I_y \ln \frac{1}{D_{xy}}\right)$$

$$= 2 \times 10^{-7}\left(I \ln \frac{1}{r'_x} - I \ln \frac{1}{D}\right)$$

$$= 2 \times 10^{-7} I \ln \frac{D}{r'_x} \quad \text{Wb-t/m}$$

where $r'_x = e^{-1/4} r_x = 0.7788 r_x$.

The inductance of conductor x is then

$$L_x = \frac{\lambda_x}{I_x} = \frac{\lambda_x}{I} = 2 \times 10^{-7} \ln \frac{D}{r'_x} \quad \text{H/m per conductor}$$

$$\lambda_y = 2 \times 10^{-7}\left(I_x \ln \frac{1}{D_{yx}} + I_y \ln \frac{1}{D_{yy}}\right)$$

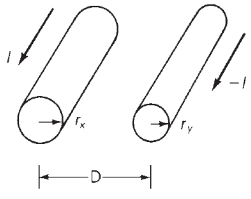
$$= 2 \times 10^{-7}\left(I \ln \frac{1}{D} - I \ln \frac{1}{r'_y}\right)$$

$$= -2 \times 10^{-7} I \ln \frac{D}{r'_y}$$

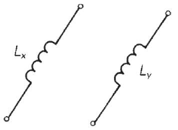
and

$$L_y = \frac{\lambda_y}{I_y} = \frac{\lambda_y}{-I} = 2 \times 10^{-7} \ln \frac{D}{r'_y} \quad \text{H/m per conductor}$$

$$\left(L = L_x + L_y = 2 \times 10^{-7}\left(\ln \frac{D}{r'_x} + \ln \frac{D}{r'_y}\right) = 2 \times 10^{-7} \ln \frac{D^2}{r'_x r'_y} = 4 \times 10^{-7} \ln \frac{D}{\sqrt{r'_x r'_y}} \quad \text{H/m per circuit}\right)$$



(a) Geometry

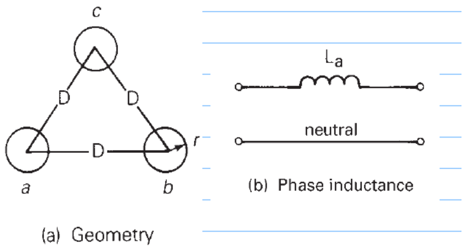


(b) Inductances

Also, if $r'_x = r'_y = r'$, the total circuit inductance is

$$L = 4 \times 10^{-7} \ln \frac{D}{r'} \quad \text{H/m per circuit}$$

② 3ϕ three-wire line with equal spacing:



(a) Geometry

(b) Phase inductance

$$\lambda_a = 2 \times 10^{-7}\left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D}\right)$$

$$= 2 \times 10^{-7}\left[I_a \ln \frac{1}{r'} + (I_b + I_c) \ln \frac{1}{D}\right]$$

Using $(I_b + I_c) = -I_a$,

$$\lambda_a = 2 \times 10^{-7}\left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D}\right)$$

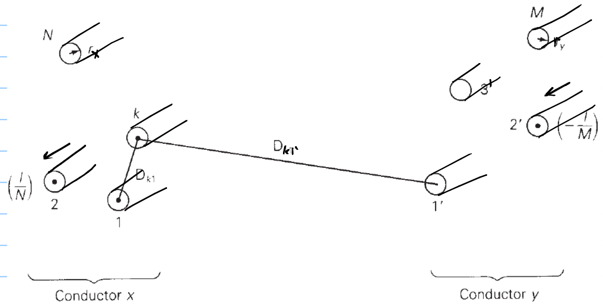
$$= 2 \times 10^{-7} I_a \ln \frac{D}{r'} \quad \text{Wb-t/m}$$

The inductance of phase a is then

$$L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \ln \frac{D}{r'} \quad \text{H/m per phase}$$, $L_b = \frac{\lambda_b}{I_b}$ and $L_c = \frac{\lambda_c}{I_c}$

4.6 Inductance: Composite conductors, unequal phase spacing, bundled conductors:

① Composite Conductors:



$$\Phi_k = 2 \times 10^{-7}\left[\frac{I}{N}\sum_{m=1}^{N} \ln \frac{1}{D_{km}} - \frac{I}{M}\sum_{m=1'}^{M} \ln \frac{1}{D_{km}}\right]$$

$$\lambda_k = \frac{\Phi_k}{N} = 2 \times 10^{-7} I\left[\frac{1}{N^2}\sum_{m=1}^{N} \ln \frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M} \ln \frac{1}{D_{km}}\right]$$

The total flux linkage of conductor x is

$$\lambda_x = \sum_{k=1}^{N} \lambda_k = 2 \times 10^{-7} I \sum_{k=1}^{N}\left[\frac{1}{N^2}\sum_{m=1}^{N} \ln \frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M} \ln \frac{1}{D_{km}}\right]$$

$$\lambda_x = 2 \times 10^{-7} I \ln \prod_{k=1}^{N} \frac{\left(\prod_{m=1'}^{M} D_{km}\right)^{1/NM}}{\left(\prod_{m=1}^{N} D_{km}\right)^{1/N^2}}$$

$$L_x = 2 \times 10^{-7} \ln \frac{D_{xy}}{D_{xx}} \quad \text{H/m per conductor}$$, where $\underset{\text{(GMD)}}{D_{xy}} = \sqrt[MN]{\prod_{k=1}^{N}\prod_{m=1'}^{M} D_{km}}$ & $\underset{\text{(GMR)}}{D_{xx}} = \sqrt[N^2]{\prod_{k=1}^{N}\prod_{m=1}^{N} D_{km}}$

$$L_y = 2 \times 10^{-7} \ln \frac{D_{xy}}{D_{yy}} \quad \text{H/m per conductor}$$, where $$D_{yy} = \sqrt[M^2]{\prod_{k=1'}^{M} \prod_{m=1'}^{M} D_{km}}$$

$$L = L_x + L_y \quad \text{H/m per circuit}$$

Solve Ex. 4.2

② Unequale phase spacing:

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_S} \quad \text{H/m}$$

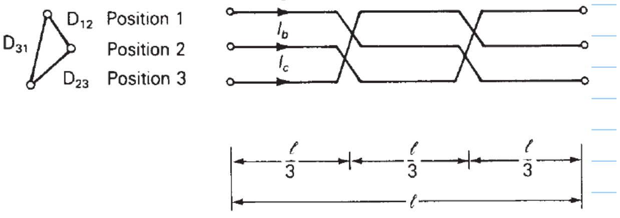
↖ GMR

where,

$$D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$$

Read Ex 4.4

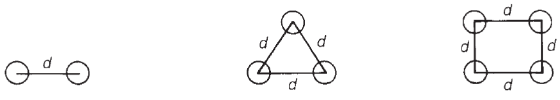
Transposition



③ Bundled conductors: (EHV)

Adv: ⓐ To reduce the electric field (corona)

ⓑ " " the series reactance of the line.



Two-conductor bundle:

$$D_{SL} = \sqrt[4]{(D_S \times d)^2} = \sqrt{D_S d}$$

Three-conductor bundle:

$$D_{SL} = \sqrt[9]{(D_S \times d \times d)^3} = \sqrt[3]{D_S d^2}$$

Four-conductor bundle:

$$D_{SL} = \sqrt[16]{(D_S \times d \times d \times d\sqrt{2})^4} = 1.091 \sqrt[4]{D_S d^3}$$

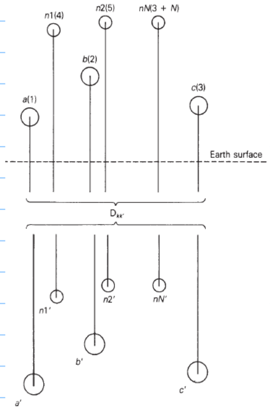
The inductance is then

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_{SL}} \quad \text{H/m}$$

Read Ex 4.5

4.7. Series Impedances: 3Ø Line w/ Neutral Conductors and Earth Return:

Type of Earth	Resistivity (Ωm)	$D_{kk'}$ (m)
Sea water	0.01 1.0	8.50 85.0
Swampy ground	10 100	269 850
Average damp earth	100	850
Dry earth	1000	2690
Pure slate	10^7	269,000
Sandstone	10^9	2,690,000



(GMR) $D_{k'k'} = D_{kk}$ m

distance $D_{kk'} = 658.5\sqrt{\rho/f}$ m

$R_{k'} = 9.869 \times 10^{-7} f$ Ω/m

$$\sum_{k=1}^{(6+2N)} I_k = 0 \implies \lambda_k = 2 \times 10^{-7} \sum_{m=1}^{(3+N)} I_m \ln \frac{D_{km'}}{D_{km}} \quad \text{Wb-t/m}$$

$\boldsymbol{\lambda} = \mathbf{L}\boldsymbol{I}$

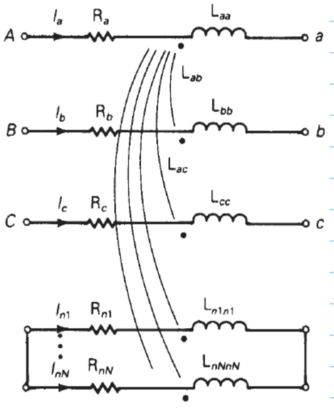
where

$\boldsymbol{\lambda}$ is a $(3+N)$ vector

$\boldsymbol{I}$ is a $(3+N)$ vector

$\mathbf{L}$ is a $(3+N) \times (3+N)$ matrix whose elements are:

$$L_{km} = 2 \times 10^{-7} \ln \frac{D_{km'}}{D_{km}}$$



(a) Complete circuit

$$\begin{bmatrix} E_{Aa} \\ E_{Bb} \\ E_{Cc} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = (\mathbf{R} + j\omega\mathbf{L}) \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_{n1} \\ \vdots \\ I_{nN} \end{bmatrix}$$

where, $$\mathbf{R} = \begin{bmatrix} (R_a + R_{k'}) R_{k'} \cdots & & & R_{k'} \\ R_{k'} (R_b + R_{k'}) R_{k'} \cdots & & & \vdots \\ & (R_c + R_{k'}) R_{k'} \cdots & & \\ & & (R_{n1} + R_{k'}) R_{k'} \cdots & \\ & \ddots & & \\ R_{k'} & & (R_{nN} + R_{k'}) & \end{bmatrix} \Omega/\text{m}$$

To Reduce the (3+N) equations:

$$\begin{array}{c} \\ \begin{bmatrix} E_{Aa}\\ E_{Bb}\\ E_{Cc}\\ \hline 0\\ \cdots\\ 0 \end{bmatrix} \end{array} \left[\begin{array}{ccc|ccc} \multicolumn{3}{c|}{\overbrace{\hphantom{Z_{(3+N)1}\quad Z_{(3+N)2}\quad Z_{(3+N)3}}}^{Z_{A(3X3)}}} & \multicolumn{3}{c}{\overbrace{\hphantom{Z_{(3+N)4}\quad \cdots\quad Z_{(3+N)(3+N)}}}^{Z_{B(3XN)}}}\\ Z_{11} & Z_{12} & Z_{13} & Z_{14} & \cdots & Z_{1(3+N)}\\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & \cdots & Z_{2(3+N)}\\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & \cdots & Z_{3(3+N)}\\ \hline Z_{41} & Z_{42} & Z_{43} & Z_{44} & \cdots & Z_{4(3+N)}\\ \\ Z_{(3+N)1} & Z_{(3+N)2} & Z_{(3+N)3} & Z_{(3+N)4} & \cdots & Z_{(3+N)(3+N)}\\ \multicolumn{3}{c|}{\underbrace{\hphantom{Z_{(3+N)1}\quad Z_{(3+N)2}\quad Z_{(3+N)3}}}_{Z_{C(NX3)}}} & \multicolumn{3}{c}{\underbrace{\hphantom{Z_{(3+N)4}\quad \cdots\quad Z_{(3+N)(3+N)}}}_{Z_{D(NXN)}}} \end{array}\right] \begin{bmatrix} I_a\\ I_b\\ I_c\\ \hline I_{n1}\\ I_{nN}\\ \vdots \end{bmatrix}$$

$$\boldsymbol{E}_P = \boldsymbol{Z}_A\boldsymbol{I}_P + \boldsymbol{Z}_B\boldsymbol{I}_n$$

$$\boldsymbol{0} = \boldsymbol{Z}_C\boldsymbol{I}_P + \boldsymbol{Z}_D\boldsymbol{I}_n$$

Solving (4.7.15) for $\boldsymbol{I}_n$,

$$\boldsymbol{I}_n = -\boldsymbol{Z}_D^{-1}\boldsymbol{Z}_C\boldsymbol{I}_P$$

Using (4.7.16) in (4.7.14):

$$\boldsymbol{E}_P = \underbrace{[\boldsymbol{Z}_A - \boldsymbol{Z}_B\boldsymbol{Z}_D^{-1}\boldsymbol{Z}_C]}_{Z_P}\boldsymbol{I}_P = \boldsymbol{Z}_P\boldsymbol{I}_P$$

⇒

The diagonal elements of this matrix are

$$Z_{kk} = \mathrm{R}_k + \mathrm{R}_{k'} + j\omega 2 \times 10 \ln\frac{\mathrm{D}_{kk'}}{\mathrm{D}_{kk}} \quad \Omega/\mathrm{m}$$

And the off-diagonal elements, for $k \neq m$, are

$$Z_{km} = \mathrm{R}_{k'} + j\omega 2 \times 10 \ln\frac{\mathrm{D}_{km'}}{\mathrm{D}_{km}} \quad \Omega/\mathrm{m}$$

Next, (4.7.10) is partitioned as shown above to obtain

$$\begin{bmatrix} \boldsymbol{E}_P\\ \hline \boldsymbol{0} \end{bmatrix} = \left[\begin{array}{c|c} \boldsymbol{Z}_A & \boldsymbol{Z}_B\\ \hline \boldsymbol{Z}_C & \boldsymbol{Z}_D \end{array}\right] \begin{bmatrix} \boldsymbol{I}_P\\ \hline \boldsymbol{I}_n \end{bmatrix}$$

where

$$\boldsymbol{E}_P = \begin{bmatrix} E_{Aa}\\ E_{Bb}\\ E_{Cc} \end{bmatrix}; \quad \boldsymbol{I}_P = \begin{bmatrix} I_a\\ I_b\\ I_c \end{bmatrix}; \quad \boldsymbol{I}_n = \begin{bmatrix} I_{n1}\\ \vdots\\ I_{nN} \end{bmatrix}$$

$$\boldsymbol{Z}_P = \begin{bmatrix} Z_{aaeq} & Z_{abeq} & Z_{aceq}\\ Z_{abeq} & Z_{bbeq} & Z_{bceq}\\ Z_{aceq} & Z_{bceq} & Z_{cceq} \end{bmatrix} \quad \Omega/\mathrm{m}$$

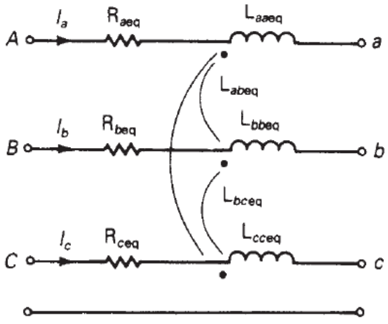
If completely transposed:

$$\hat{\boldsymbol{Z}}_P = \begin{bmatrix} \hat{Z}_{aaeq} & \hat{Z}_{abeq} & \hat{Z}_{abeq}\\ \hat{Z}_{abeq} & \hat{Z}_{aaeq} & \hat{Z}_{abeq}\\ \hat{Z}_{abeq} & \hat{Z}_{abeq} & \hat{Z}_{aaeq} \end{bmatrix} \quad \Omega/\mathrm{m}$$

where

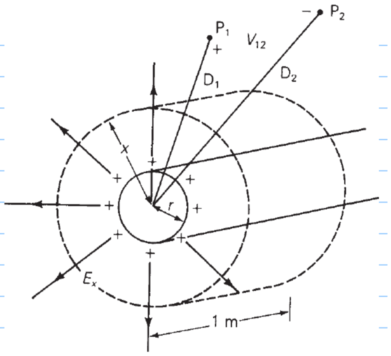
$$\hat{Z}_{aaeq} = \tfrac{1}{3}(Z_{aaeq} + Z_{bbeq} + Z_{cceq})$$

$$\hat{Z}_{abeq} = \tfrac{1}{3}(Z_{abeq} + Z_{aceq} + Z_{bceq})$$



(b) Simplified circuit

4.8 Electric Field & Voltage: Solid Cylendrical Conductor:



The capacitance between conductors:

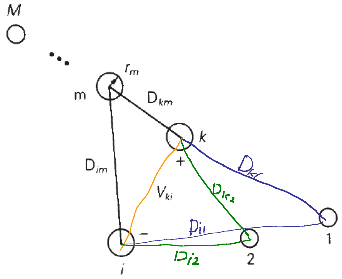
1. Electric field strength E, from Gauss's law
2. Voltage between conductors
3. Capacitance from charge per unit volt ($C = q/V$)

① The voltage between two points outside the conductor:

Gauss's Law: $\oiint \underset{\substack{\uparrow\\ \text{density}}}{D_\perp}\, ds = \oiint \varepsilon \underset{\substack{\uparrow\\ \text{strength}}}{E_\perp}\, ds = Q_{\text{enclosed}} \Rightarrow \varepsilon E_x(2\pi x)(1) = q(1)$, rearrange: $E_x = \dfrac{q}{2\pi\varepsilon x}$ V/m

$$V_{12} = \int_{D_1}^{D_2} E_x\, dx = \int_{D_1}^{D_2} \frac{q}{2\pi\varepsilon x}\, dx = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1} \quad \text{volts}$$

② Voltage between two conductors in an array of charged conductors:



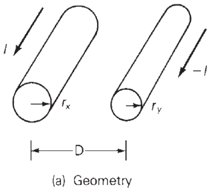
$$V_{\underset{\substack{\swarrow\\ V_k}}{k}\underset{\substack{\swarrow\\ V_i}}{i}\underset{\substack{\downarrow\\ q_M}}{m}} = \frac{q_m}{2\pi\varepsilon} \ln \frac{D_{im}}{D_{km}} \quad \text{volts}$$

Using superposion:

$$V_{ki} = \frac{1}{2\pi\varepsilon} \sum_{m=1}^{M} q_m \ln \frac{D_{im}}{D_{km}} \quad \text{volts}, \quad V_{ki} = V_{ki1} + V_{ki2} + \cdots + V_{kiM}$$

4.9 Capacetance: 1∅ two-wire Line & 3∅ three-wire Line w/ equal ∅ spacing:

① 1∅ two-wire Line:



(a) Geometry

$$V_{xy} = \frac{1}{2\pi\varepsilon}\left[q \ln \frac{D_{yx}}{D_{xx}} - q \ln \frac{D_{yy}}{D_{xy}}\right],$$

$$= \frac{q}{2\pi\varepsilon} \ln \frac{D_{yx}D_{xy}}{D_{xx}D_{yy}},$$

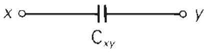
Using $D_{xy} = D_{yx} = D$, $D_{xx} = r_x$, and $D_{yy} = r_y$

$$\Rightarrow V_{xy} = \frac{q}{\pi\varepsilon} \ln \frac{D}{\sqrt{r_x r_y}} \quad \text{volts}$$

✳ For a 1-meter line length, the capacitance between conductors is

$$C_{xy} = \frac{q}{V_{xy}} = \frac{\pi\varepsilon}{\ln\left(\dfrac{D}{\sqrt{r_x r_y}}\right)} \quad \text{F/m line-to-line},$$

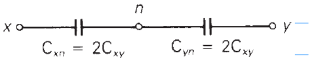
and if $r_x = r_y = r$, $\left(C_{xy} = \dfrac{\pi\varepsilon}{\ln(D/r)} \quad \text{F/m line-to-line}\right)$



(a) Line-to-line capacitance

∴ If the two-wire line is supplied by a transformer with a grounded center tap,

$$C_n = C_{xn} = C_{yn} = \frac{q}{V_{xn}} = 2C_{xy} = \frac{2\pi\varepsilon}{\ln(D/r)} \quad \text{F/m line-to-neutral}$$

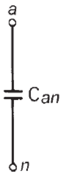


(b) Line-to-neutral capacitances

② 3∅ three-wire line: (Equal phase spacing)

Assume $q_a + q_b + q_c = 0$,

Using $D_{aa} = D_{bb} = r$, and $D_{ab} = D_{ba} = D_{ca} = D_{cb} = D$,



$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D_{ba}}{D_{aa}} + q_b \ln \frac{D_{bb}}{D_{ab}} + q_c \ln \frac{D_{bc}}{D_{ac}}\right] = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D}{r} + q_b \ln \frac{r}{D} + q_c \ln \frac{D}{D}\right] = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D}{r} + q_b \ln \frac{r}{D}\right] \quad \text{volts}$$

similarly, $V_{ac} = \dfrac{1}{2\pi\varepsilon}\left[q_a \ln \dfrac{D}{r} + q_c \ln \dfrac{r}{D}\right]$ volts

$$\left.\begin{aligned} V_{ab} &= \sqrt{3}V_{an}\underline{/+30^\circ} = \sqrt{3}V_{an}\left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right] \\ V_{ac} &= -V_{ca} = \sqrt{3}V_{an}\underline{/-30^\circ} = \sqrt{3}V_{an}\left[\frac{\sqrt{3}}{2} - j\frac{1}{2}\right] \\ q_b + q_c &= -q_a \end{aligned}\right\} V_{ab} + V_{ac} = 3V_{an} \Longrightarrow V_{an} = \frac{1}{3}\left(\frac{1}{2\pi\varepsilon}\right)\left[2q_a \ln \frac{D}{r} + (q_b + q_c) \ln \frac{r}{D}\right] = \frac{1}{2\pi\varepsilon} q_a \ln \frac{D}{r} \quad \text{volts}$$

$$\therefore C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{\ln\left(\dfrac{D}{r}\right)} \quad \text{F/m line-to-neutral}$$

4.10 Capacitance: Stranded Conductors, Unequal ϕ spacing, Bundled Conductors:

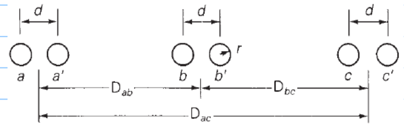
① Transposed:

$$C_{an} = \frac{2\pi\varepsilon}{\ln(D_{eq}/r)} \quad \text{F/m}$$

where

$$D_{eq} = \sqrt[3]{D_{ab}D_{bc}D_{ac}}$$

② Bundled & Transposed:



$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[\frac{q_a}{2}\ln\frac{D_{ba}}{D_{aa}} + \frac{q_a}{2}\ln\frac{D_{ba'}}{D_{aa'}} + \frac{q_b}{2}\ln\frac{D_{bb}}{D_{ab}} + \frac{q_b}{2}\ln\frac{D_{bb'}}{D_{ab'}} + \frac{q_c}{2}\ln\frac{D_{bc}}{D_{ac}} + \frac{q_c}{2}\ln\frac{D_{bc'}}{D_{ac'}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[\frac{q_a}{2}\left(\ln\frac{D_{ab}}{r} + \ln\frac{D_{ab}}{d}\right) + \frac{q_b}{2}\left(\ln\frac{r}{D_{ab}} + \ln\frac{d}{D_{ab}}\right) + \frac{q_c}{2}\left(\ln\frac{D_{bc}}{D_{ac}} + \ln\frac{D_{bc}}{D_{ac}}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{ab}}{\sqrt{rd}} + q_b \ln\frac{\sqrt{rd}}{D_{ab}} + q_c \ln\frac{D_{bc}}{D_{ac}}\right]$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln(\underset{\text{Transpose}}{D_{eq}}/\underset{\text{Bundled}}{D_{SC}})} \quad \text{F/m}$$

where

$$D_{SC} = \sqrt{rd} \quad \text{for a two-conductor bundle}$$

Similarly,

$$D_{SC} = \sqrt[3]{rd^2} \quad \text{for a three-conductor bundle}$$

$$D_{SC} = 1.091\sqrt[4]{rd^3} \quad \text{for a four-conductor bundle}$$

---

* Charging Current:

① 1ϕ:

L-L voltage ⇒ $V_{xy} = V_{xy}\underline{/0°}$

$$\therefore\ I_{chg} = Y_{xy}V_{xy} = j\omega C_{xy}V_{xy} \quad \text{A}$$

$$Q_C = \frac{V_{xy}^2}{X_c} = Y_{xy}V_{xy}^2 = \omega C_{xy}V_{xy}^2 \quad \text{var}$$ delivered by the L-L capacitance.

② 3ϕ:

L-N voltage ⇒ $V_{an} = V_{LN}\underline{/0°}$

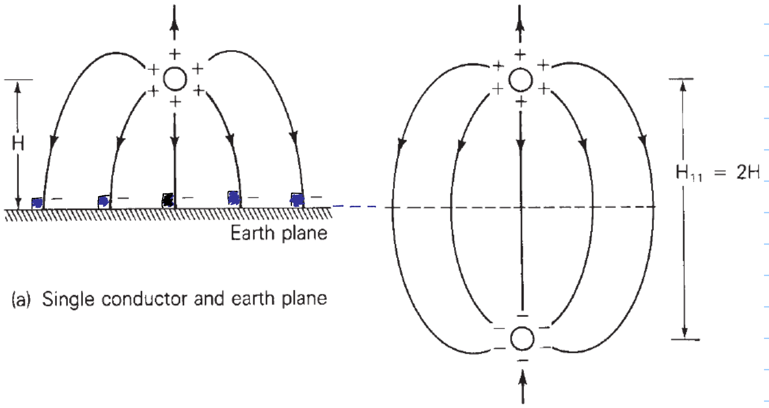
$$\therefore\ I_{chg} = YV_{an} = j\omega C_{an}V_{LN} \quad \text{A}$$

Read Ex 4.6 & 4.7

$$Q_{C1\phi} = YV_{an}^2 = \omega C_{an}V_{LN}^2 \quad \text{var}$$ delivered by phase a

$$Q_{C3\phi} = 3Q_{C1\phi} = 3\omega C_{an}V_{LN}^2 = \omega C_{an}V_{LL}^2 \quad \text{var}$$ the total var supplied by the 3ϕ line

4.11 Shunt Admittances: Lines with Neutral conductors and Earth Returns:



(a) Single conductor and earth plane

(b) Earth plane replaced by image conductor

Solve Ex 4.8

$$V_{kk'} = \frac{1}{2\pi\varepsilon}\left[\underbrace{\sum_{m=a}^{nN} q_m \ln\frac{H_{km}}{D_{km}}}_{\text{above earth plane}} - \underbrace{\sum_{m=a}^{nN} q_m \ln\frac{D_{km}}{H_{km}}}_{\text{under earth plane}}\right] = \frac{2}{2\pi\varepsilon}\sum_{m=a}^{nN} q_m \ln\frac{H_{km}}{D_{km}}$$

$D_{kk} = r_k$

$$V_{kn} = \frac{1}{2}V_{kk'} = \frac{1}{2\pi\varepsilon}\sum_{m=a}^{nN} q_m \ln\frac{H_{km}}{D_{km}}$$

because all neutrals are grounded to earth

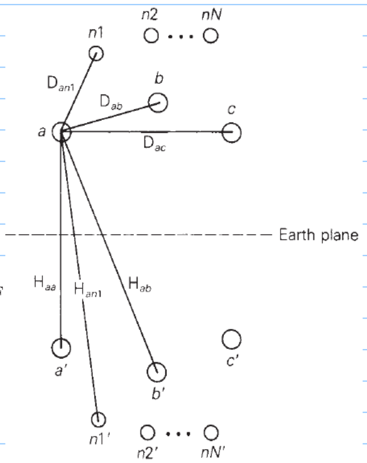
$$\begin{bmatrix} V_{an}\\ V_{bn}\\ V_{cn}\\ \hline 0\\ \vdots\\ 0 \end{bmatrix} = \left[\begin{array}{ccc|ccc} \multicolumn{3}{c|}{\mathbf{P}_{A\,3\times3}} & \multicolumn{3}{c}{\mathbf{P}_{B\,3\times N}} \\ P_{aa} & P_{ab} & P_{ac} & P_{an1} & \cdots & P_{anN} \\ P_{ba} & P_{bb} & P_{bc} & P_{bn1} & \cdots & P_{bnN} \\ P_{ca} & P_{cb} & P_{cc} & P_{cn1} & \cdots & P_{cnN} \\ \hline P_{n1a} & P_{n1b} & P_{n1c} & P_{n1n1} & \cdots & P_{n1nN} \\ \vdots & & & & & \\ P_{nNa} & P_{nNb} & P_{nNc} & P_{nNn1} & \cdots & P_{nNnN} \\ \multicolumn{3}{c|}{\mathbf{P}_{C\,N\times3}} & \multicolumn{3}{c}{\mathbf{P}_{D\,N\times N}} \end{array}\right] \begin{bmatrix} q_a\\ q_b\\ q_c\\ \hline q_{n1}\\ \vdots\\ q_{nN} \end{bmatrix} \Rightarrow$$

$$P_{km} = \frac{1}{2\pi\varepsilon}\ln\frac{H_{km}}{D_{km}} \quad \text{m/F}$$

where

$k = a, b, c, n1, \ldots, nN$

$m = a, b, c, n1, \ldots, nN$



$$\left[\frac{V_P}{0}\right] = \left[\begin{array}{c|c} P_A & P_B \\ \hline P_C & P_D \end{array}\right]\left[\frac{q_P}{q_n}\right]$$

$$\left.\begin{aligned} V_P &= P_A q_P + P_B q_n \\ 0 &= P_C q_P + P_D q_n \end{aligned}\right\} \quad V_P = \overbrace{(P_A - P_B P_D^{-1} P_C)}^{C_P^{-1}} q_P \Rightarrow q_P = C_P V_P$$

where,

$$C_P = \begin{bmatrix} C_{aa} & C_{ab} & C_{ac} \\ C_{ab} & C_{bb} & C_{bc} \\ C_{ac} & C_{bc} & C_{cc} \end{bmatrix} \quad \text{F/m}$$

(+) (−) (−)

The shunt phase admittance matrix is given by

$$Y_P = j\omega C_P = j(2\pi f)C_P \quad \text{S/m}$$

or, for a completely transposed line,

$$\hat{Y}_P = j\omega\hat{C}_P = j(2\pi f)\hat{C}_P \quad \text{S/m}$$

If the line is completely transposed:

$$\hat{C}_P = \begin{bmatrix} \hat{C}_{aa} & \hat{C}_{ab} & \hat{C}_{ab} \\ \hat{C}_{ab} & \hat{C}_{aa} & \hat{C}_{ab} \\ \hat{C}_{ab} & \hat{C}_{ab} & \hat{C}_{aa} \end{bmatrix} \quad \text{F/m}$$

where,

$$\hat{C}_{aa} = \tfrac{1}{3}(C_{aa} + C_{bb} + C_{cc}) \quad \text{F/m}$$

$$\hat{C}_{ab} = \tfrac{1}{3}(C_{ab} + C_{bc} + C_{ac}) \quad \text{F/m}$$

4.12 Electric Field Strenght at Conductor Surfaces & at Ground Level:

Corona: Line losses caused by an excess in the electric field strength at a conductor surface. >30kV/cm

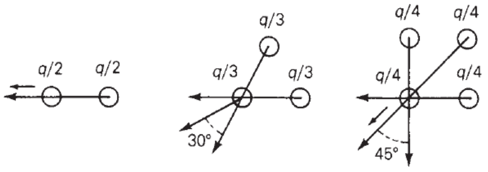
* The electric field strength on one conductor:

$$E_r = \frac{q}{2\pi\varepsilon r} \quad \text{V/m}$$

* If bundled:

$$E_{rave} = \frac{q/N_b}{2\pi\varepsilon r} \quad \text{V/m}$$

, N_b is # of conductors/phase.



Two-conductor bundle ($N_b = 2$):

* $$E_{r\max} = \frac{q/2}{2\pi\varepsilon r} + \frac{q/2}{2\pi\varepsilon d} = \frac{q/2}{2\pi\varepsilon r}\left(1 + \frac{r}{d}\right)$$

$$= E_{rave}\left(1 + \frac{r}{d}\right)$$

Three-conductor bundle ($N_b = 3$):

* $$E_{r\max} = \frac{q/3}{2\pi\varepsilon}\left(\frac{1}{r} + \frac{2\cos 30^\circ}{d}\right) = E_{rave}\left(1 + \frac{r\sqrt{3}}{d}\right)$$

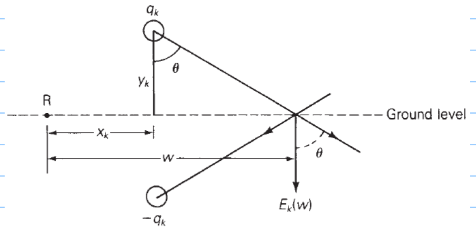
Four-conductor bundle ($N_b = 4$):

* $$E_{r\max} = \frac{q/4}{2\pi\varepsilon}\left(\frac{1}{r} + \frac{1}{d\sqrt{2}} + \frac{2\cos 45^\circ}{d}\right) = E_{rave}\left[1 + \frac{r}{d}(2.1213)\right]$$

* Ground-Level electric field strength:

$$E_k(w) = \left(\frac{q_k}{2\pi\varepsilon}\right)\frac{2\cos\theta}{\sqrt{y_k^2+(w-x_k)^2}}$$

$$= \left(\frac{q_k}{2\pi\varepsilon}\right)\frac{2y_k}{y_k^2+(w-x_k)^2}\quad \text{V/m}$$

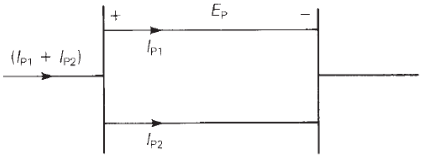


Line Voltage (kV_{rms})	Maximum Ground-Level Electric Field Strength (kV_{rms}/m)
23 (1ϕ)	0.01–0.025
23 (3ϕ)	0.01–0.05
115	0.1–0.2
345	2.3–5.0
345 (double circuit)	5.6
500	8.0
765	10.0

Solve Ex 4.9

4.13 Parallel Circuit Three-phase Line:

* Impedance:



$$\begin{bmatrix} \boldsymbol{E}_P \\ \boldsymbol{E}_P \end{bmatrix} = \boldsymbol{Z}_P \begin{bmatrix} \boldsymbol{I}_{P1} \\ \boldsymbol{I}_{P2} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{I}_{P1} \\ \hline \boldsymbol{I}_{P2} \end{bmatrix} = \underset{(6\times 6)}{\boldsymbol{Z}_P^{-1}} \begin{bmatrix} \boldsymbol{E}_P \\ \hline \boldsymbol{E}_P \end{bmatrix} = \left[\begin{array}{c|c} \boldsymbol{Y}_A & \boldsymbol{Y}_B \\ \hline \boldsymbol{Y}_C & \boldsymbol{Y}_D \end{array}\right] \begin{bmatrix} \boldsymbol{E}_P \\ \hline \boldsymbol{E}_P \end{bmatrix} = \begin{bmatrix} (\boldsymbol{Y}_A+\boldsymbol{Y}_B) \\ (\boldsymbol{Y}_C+\boldsymbol{Y}_D) \end{bmatrix} \boldsymbol{E}_P$$

$$(\boldsymbol{I}_{P1}+\boldsymbol{I}_{P2}) = \underbrace{(\boldsymbol{Y}_A+\boldsymbol{Y}_B+\boldsymbol{Y}_C+\boldsymbol{Y}_D)}_{Z_{Peq}^{-1}}\boldsymbol{E}_P \Rightarrow \boldsymbol{E}_P = \underset{(3\times 3)}{\boldsymbol{Z}_{Peq}}(\boldsymbol{I}_{P1}+\boldsymbol{I}_{P2}) \Rightarrow \boldsymbol{Z}_{Peq} = (\boldsymbol{Y}_A+\boldsymbol{Y}_B+\boldsymbol{Y}_C+\boldsymbol{Y}_D)^{-1}$$

* Shunt Admittance:

$$\begin{bmatrix} \boldsymbol{q}_{P1} \\ \hline \boldsymbol{q}_{P2} \end{bmatrix} = \mathbf{C}_P \begin{bmatrix} \boldsymbol{V}_P \\ \hline \boldsymbol{V}_P \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{C}_A & \mathbf{C}_B \\ \hline \mathbf{C}_C & \mathbf{C}_D \end{array}\right] \begin{bmatrix} \boldsymbol{V}_P \\ \hline \boldsymbol{V}_P \end{bmatrix} = \begin{bmatrix} (\mathbf{C}_A+\mathbf{C}_B) \\ (\mathbf{C}_C+\mathbf{C}_D) \end{bmatrix} \boldsymbol{V}_P$$

$(\boldsymbol{q}_{P1}+\boldsymbol{q}_{P2}) = \mathbf{C}_{Peq}\boldsymbol{V}_P$, where, $\mathbf{C}_{Peq} = (\mathbf{C}_A+\mathbf{C}_B+\mathbf{C}_C+\mathbf{C}_D) \Rightarrow \boldsymbol{Y}_{Peq} = j\omega\mathbf{C}_{Peq}$